Instructions

- Check your name: Shashi Kant
- Check your roll #: 5010137

Section N

NQ 1 Let X be a random variable whose variance is non–zero. Set $Y = 3 - 2X$. Find the correlation $\rho(X, Y)$ between X and Y.

NQ 2 Let Z be the standard normal random variable. Define a new random variable

$$X = \begin{cases} Z & \text{if } Z > \sqrt{2} \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X)$.

NQ 3 We have 100 lightbulbs whose lifetimes are independent exponential random variables with mean 5 hours. The bulbs are used one at a time, with a failed bulb being replaced immediately by a new one. What is the probability that there is still a working bulb after 555 hours?

NQ 4 Suppose the joint density of random variables X_1 and X_2 is uniform over the rectangle $(0, 2) \times (0, 4)$ in $\mathbb{R}^2$ (while being zero outside the stated rectangle). Define random variables Y_1 and Y_2 via $Y_1 = 2X_1 + X_2$ and $Y_2 = X_1 + 3X_2$. Then, the joint density of Y_1 and Y_2 is uniform over a certain quadrilateral with a value c (while being zero outside the stated quadrilateral). What is c?

NQ 5 In MA225, suppose your 'InSem' marks is a normal random variable with mean 30 and variance 16 while your 'EndSem' marks is a normal random variable with mean 20 and variance 9 and these two are independent. Your instructor plans to give you an AA if your total is at least 62 marks. What is the probability of you getting an AA in MA225? (You should include continuity correction)

NQ 6 Suppose X and Y are independent gamma random variables with parameters $\left(\frac{2}{3}, 3\right)$ and $\left(\frac{4}{3}, 3\right)$. Find $P(X + Y \geq 1)$.

NQ 7 Let $X(n)$ for $n \in \mathbb{N}$ represent a Markov chain such that for each n, $X(n)$ can be in one of the three states numbered $1, 2, 3$. Suppose further that for all $n \geq 2$, $P(X(n) = j | X(n-1) = i) = p_{ij}$ for $i, j \in \{1, 2, 3\}$ where the matrix P is as given below:

$$\begin{pmatrix} .2 & .3 & \alpha \\ .3 & .4 & \beta \\ .4 & .5 & \gamma \end{pmatrix},$$

for some real numbers α, β and γ. Find $\alpha + \beta + \gamma$.

NQ 8 A fair die is successively rolled. Let X and Y denote respectively the number of rolls necessary to obtain a 6 and a 5. Find $E[X|Y = 1]$.

NQ 9 The joint probability density of random variables X and Y is given by

$$f(x, y) = \begin{cases} e^{-x-y} & \text{if } 0 \leq x < \infty \quad \& \quad 0 \leq y < \infty \\ 0 & \text{otherwise.} \end{cases}$$

Find $P(X < 2)$.

NQ 10 X is binomial with $n = 10,000$ and $p = 0.2$. Applying the de-Moivre–Laplace Theorem, find the approximate value of $P(1955 \leq X \leq 2090)$. (You should include continuity correction)

NQ 11 For $i = 1, 2, \ldots, 5$ consider the i-th trial which results in success with a probability of $\frac{1}{3^i}$. What is the expected value of the total number of successes in these five trials?

NQ 12 Let X, Y and Z be independent exponential random variables with parameters $2, 3$ and 4 respectively. Find $P(X < Y < Z)$.

Section J

JQ 13 (Jackpot: 10 marks for full solution, NO partial credit.) Let X and Y be discrete random variables taking the values $1, 2, 3, \ldots$. Suppose the moment generating functions of X and Y are equal. Show that $X = Y$.

Space For Answers & Not For Rough Work

Q#:

Instructions

- Check that you have 4 printed pages for this test.
- Use a Black or Blue pen to bubble your answers for Questions 1–12 on Sheet B (Page #4), not in the answer booklet. The right way to fill a bubble like ☐ is as in ■.
- Please note that the options A, B, etc. are given vertically/horizontally in the booklet whereas they are printed horizontally on sheet B.
- Section N contains questions for which a numerical value is expected as the answer. Suppose the answer to such a question is $a = \pm\beta$ and suppose there are only k decimal places printed for this question on the bubble sheet. Then you should bubble $\pm$ accordingly as a is positive or negative, bubble round(β, d) where d is the number of decimal places for which boxes are printed on the bubble sheet for the question under consideration. For example round(2.341,2)=2.34; round(2.345,2)=2.35; round(2.347,2)=2.35; round$(-2.341, 2) = -2.34$; round$(-2.345, 2) = -2.35$ etc. Note that the rounding has to be done only for the final answer and not intermediate steps.
- Use supplementary sheets only for rough work.

Test pattern and Scoring scheme:

Section	Type	Questions	Marks	Grading
N	Numeric Answer	1–12	1	0, .9 or 1
J	Jackpot	13–13	10	0 or 10

Table of standard normal distribution:

x	0.0000	0.0150	0.0275	0.0375	0.0500	0.0625	0.0750	0.0875	0.1000	0.1125
1.1	0.86433	0.86757	0.87023	0.87234	0.87493	0.87748	0.88000	0.88248	0.88493	0.88734
2.1	0.98214	0.98279	0.98331	0.98372	0.98422	0.98471	0.98518	0.98565	0.98610	0.98653
2.2	0.98610	0.98662	0.98704	0.98737	0.98778	0.98817	0.98855	0.98892	0.98928	0.98962
2.3	0.98928	0.98969	0.99003	0.99029	0.99061	0.99092	0.99123	0.99152	0.99180	0.99208
2.4	0.99180	0.99213	0.99240	0.99261	0.99286	0.99310	0.99334	0.99357	0.99379	0.99401

Do not write anything on this page

Bubble Sheet B

Check your details and sign:

Name : Shashi Kant
Roll # : 5010137
Venue : 2201

Student's Sign. :

Invigilator's Sign. :

Bubble Your Answers Below

NQ 1:

	0	1	2	3	4	5	6	7	8	9
	•									
+	0	1	2	3	4	5	6	7	8	9
−	0	1	2	3	4	5	6	7	8	9

NQ 2:

•									
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 3:

•									
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 4:

0	1	2	3	4	5	6	7	8	9
•									
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 5:

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0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 6:

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0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 7:

0	1	2	3	4	5	6	7	8	9
•									
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 8:

0	1	2	3	4	5	6	7	8	9
•									
0	1	2	3	4	5	6	7	8	9

NQ 9:

•									
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 10:

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0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 11:

0	1	2	3	4	5	6	7	8	9
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0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

NQ 12:

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0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

JQ 13: ..... *Reserved* 0 10